

A Universal Formula for the Stress–Tensor Contribution to Scalar Four–Point Functions

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Abstract

We illustrate the power and efficiency of a recently uncovered Mellin–space approach to AdS/CFT correlation functions by providing a universal formula for the 4–scalar graviton–exchange Witten diagram, for arbitrary CFT–dual scaling dimensions. Our result keeps the space–time dimension generic as well, and is expressed as a combination of just 11 hypergeometric functions. Such hypergeometric functions are related to scalar–exchange diagrams. In particular, if reverse–engineered in terms of D–functions, this might be viewed as a first–step towards proving a long–standing conjecture by Dolan, Nirschl and Osborn pertaining to four–point correlators of chiral primary operators at strong coupling. Most importantly, the technology developed herein marks an additional development towards the long–anticipated computation of the four–point function of the $\mathcal{N} = 4$ sYM stress–tensor.

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1 Introduction

According to the gauge/gravity duality, supergravity scattering amplitudes in a $d + 1$ -dimensional AdS_{d+1} background encode the planar contribution to correlation functions of operators of a dual conformal field theory CFT_d at strong coupling. Yet, despite more than a decade of efforts, the evaluation of AdS/CFT correlation functions was until very recently a difficult and computationally intensive task.

Such calculations were typically done in position space. It is natural to wonder if this is the right language. In the same way that going to momentum space simplifies the evaluation of Feynman diagrams in flat space, one might suspect that, similarly, different variables could possibly be lurking around, which would facilitate the computation of Witten diagrams.

Indeed, it has recently been illustrated through various examples that applying a Mellin-transform to AdS/CFT correlators dispenses with a lot of hard work and sheds light on their structure [1, 2, 3, 4, 5]. In particular, the contents of operator product expansions is disentangled after going to Mellin space. This boils down to the fact that Mellin amplitudes are meromorphic with simple poles whose positions correspond to the scaling dimensions of operators in the OPE [6, 7]. In a sense, going to Mellin space realizes a natural basis of OPE, as first envisioned by Polyakov [8].

So far, the Mellin-space approach has mostly focused on evaluating Witten diagrams involving scalar fields only. Feynman-like rules for tree-level Mellin amplitudes have been proven [9] in this case. The purpose of this short note is to show that Mellin-space is just as well apposite to handling with great efficiency processes involving the exchange of a graviton in the bulk. We managed to find a universal formula that captures at one fell swoop the stress-tensor exchange contribution to any 4-point function of scalar operators, no matter the particular values of their conformal weights or of the space-time they live in. This should be contrasted with the great amount of effort that is exacted by a position-space approach to evaluating such diagrams, not to mention that this *modus operandi* cannot achieve the universality of our formula [10, 11, 12, 13, 14, 15, 16].

It bears noting that the validity of our formula is rigorously established by setting all the operator dimensions to $\Delta_i = \Delta = d$ in $d = 4$ dimensions, and comparing with equation (8) from [3], which is equation (44) of the present publication.

Three-point functions of chiral primary operators are determined by superconformal invariance. Therefore, their supergravity computation does not tell much¹, beyond providing further evidence for the validity of the AdS/CFT correspondence, which, of course, is beyond doubt by now. On the other hand, four-point functions of chiral primaries are not entirely constrained by symmetries or non-renormalization theorems. As such, studying their structure on the supergravity side is a challenging task that will disclose valuable information on CFT's at strong coupling.

Although we do not provide all the details of our computation², we have sought to explain a few non-trivial steps in the remainder of this note. We are especially highlighting aspects of our computation that are distinctive of processes involving the exchange of gravitons in the bulk, and are not part of the standard tool-box of previous Mellin-space computations.

¹As expected, this agrees with a free-field computation [17].

²Our Mathematica code is available upon request.

$$D_{\Delta_1 \Delta_3 \Delta_2 \Delta_4} = \text{Diagram}$$

Figure 1: *The D-function [10] for dual operators of respective conformal weight Δ_1 , Δ_2 , Δ_3 and Δ_4 .*

The last section illustrates our general formula on a few particular examples. A Mathematica notebook containing our universal formula is up for grabs from the source of the arXiv version of the present publication. This formula is somewhat lengthy but simplifies tremendously whenever arbitrary values of the operator scaling dimensions are specified. It is also worth pointing out that deriving this formula is an entirely algebraic and automated task. Former approaches would traditionally have called on performing two complicated integrations over AdS_{d+1} or solving recursively some differential equations [18].

It would be interesting to try and reverse-engineer our expression to position-space as a combination of D-functions. D-functions [10] are tantamount to tree-level n -point contact interactions between scalars in AdS (cf. Figure 1). It is true more generally that for the trilinear couplings stemming from $\text{AdS}_5 \times S^5$ supergravity, any exchange diagram reduces to a finite sum of scalar quartic graphs [18]. However, a conjecture of Dolan, Nirschl and Osborn [19] asserts that it should be possible to cast as a very particular and constrained sum of D-functions the dynamical part of four-point functions of single-trace chiral primaries belonging to any $[0, p, 0]$ $SU(4)$. This proposal for a specific combination of D-functions proceeds from an observation concerning the contribution of long multiplets with twist less than $2p$. We are aware of at least one proposal for proving this conjecture³, which relies on bootstrapping arguments. Our result would provide a cross-check and lend further credence to such an attempt, were the latter be one day brought about with success.

Last but not least, the computation herein might also be viewed as one step closer to computing the long-sought four-point function of the $\mathcal{N} = 4$ sYM stress-tensor, a demanding goal on which we comment in Section 3.

³We are grateful to Leonardo Rastelli for explanations on this undertaking.

2 A review of the Mellin–transform of Witten diagrams

Given a correlation function of n scalar primary operators of conformal weights Δ_i , $i = 1, \dots, n$, living in a d -dimensional space,

$$\mathcal{A}(x_1, \dots, x_n) \equiv \langle \mathcal{O}(x_1) \dots \mathcal{O}(x_n) \rangle, \quad (1)$$

its Mellin–transform $\mathcal{M}(\delta_{ij})$ is defined as follows:

$$\begin{aligned} \mathcal{A}(x_1, \dots, x_n) &= \frac{\mathcal{N}}{2\pi i} \int d\delta_{ij} \mathcal{M}(\delta_{ij}) \prod_{i \leq j}^n \Gamma[\delta_{ij}] (x_i - x_j)^{-2\delta_{ij}}, \\ \mathcal{N} &\equiv \frac{\pi^{d/2}}{2} \Gamma\left[\frac{\sum_{i=1}^n \Delta_i - d}{2}\right] \prod_{i=1}^n \frac{1}{2\pi^{d/2} \Gamma\left[\Delta_i - \frac{d}{2} + 1\right]}. \end{aligned} \quad (2)$$

The parameters $\delta_{ij} \equiv \delta_{ji}$ are such that, for each index i , we have

$$\sum_{j=1}^n \delta_{ij} = \Delta_i, \quad \delta_{ii} = 0. \quad (3)$$

The integration contours for the $\frac{n(n-3)}{2}$ independent integration variables above are chosen parallel to the imaginary axis and such that the real parts of the arguments of the gamma functions entering the integrand are positive.

In keeping with the idea of an effective conformal field theory for the low-lying spectrum of the dilation operator of an CFT [20, 21, 22, 23], it is illuminating to think of the δ_{ij} ’s as related to a sort of Mandelstam invariants built out of the “momentum” vectors k_i for states whose invariant masses are actually the operator dimensions Δ_i :

$$-k_i^2 \equiv \Delta_i, \quad \sum_{i=1}^n k_i \equiv 0, \quad \delta_{ij} \equiv k_i \cdot k_j \quad (4)$$

and

$$s_{i_1 \dots i_\ell} \equiv -\left(\sum_{i_m=1}^{\ell} k_{i_m}\right)^2, \quad (5)$$

so that in particular

$$s_{ij} = -(k_i + k_j)^2 = \Delta_i + \Delta_j - 2\delta_{ij}. \quad (6)$$

This interpretation is especially apt since, as it turns out, the Mellin amplitude of a given Witten diagram is meromorphic with simple poles at s_{ij} . As such, it can be thought of as a natural basis for operator product expansions. Another catchword encapsulates a Mellin–transform as a kind of Fourier transform in the radial directions $|x_i - x_j|$. Those distances typically appear in the OPE of two operators inserted at x_i and x_j .

The Mellin–transform can be put to good use in a variety of situations, as illustrated a little while back [24] in the context of finding relations between some of the multi-loop integrals of interest to QCD-like theories. Here, our main focus revolves around computing

Witten diagrams. According to the holographic dictionary, such scattering processes in the bulk of a weakly-curved asymptotically AdS_{d+1} space translate into correlation functions for the operators of a dual strongly-coupled CFT_d .

In this setting, it is especially helpful to combine the Mellin-transform with the so-called embedding formalism [25, 26, 27, 28, 29]. This simply amounts to taking advantage of the fact that an AdS_{d+1} space can be viewed as an hyperboloid embedded in an $d+2$ -dimensional Minkowski space \mathbb{M}^{d+2} with light-cone metric

$$ds^2 = -dX^+ dX^- + \sum_{m,n=1}^d \delta_{mn} dX^m dX^n \equiv \eta_{MN} dX^M dX^N. \quad (7)$$

The boundary of AdS_{d+1} corresponds to the set of null rays:

$$\{P \in \mathbb{M}^{d+2} : P^2 = 0, P \sim \lambda P, \lambda \in \mathbb{R}\}. \quad (8)$$

We can parametrize a vector P describing the boundary of AdS_{d+1} as

$$P = (1, x^2, x^\mu), \quad (9)$$

which makes contact with the more conventional description. In particular, note that

$$P_{ij} \equiv -2 P_i \cdot P_j = (x_i - x_j)^2. \quad (10)$$

This quantity shows up in the definition of a generic Mellin-transform (2).

The key advantage of the embedding formalism is that it linearizes the action of the conformal group. We have implemented this property in our calculation of an universal formula for the stress-tensor contribution to a generic Green's function of four scalar primaries at strong coupling. This involves evaluating a Witten diagram connecting two pairs of massive or massless scalars via a graviton in the bulk of AdS. The two vertices involve covariant derivatives in AdS. Quite nicely, this translates into simple partial derivatives in the flat Minkowski embedding space.

Many more details can be found in [4], whose conventions we are using.

3 The graviton bulk-to-bulk propagator

In position space, the physical part of the graviton bulk-to-bulk propagator between a point z in AdS_{d+1} and another point of coordinates w is given by

$$G_{AB;A'B'}(z, w) = [\partial_A \partial_{A'} u \partial_B \partial_{B'} u + \partial_A \partial_{B'} u \partial_B \partial_{A'} u] G(u) + g_{AB} g_{A'B'} H(u). \quad (11)$$

With the following AdS_{d+1} metric

$$ds^2 = \frac{1}{z_0^2} \left(dz_0^2 + \sum_{i=1}^d dz_i^2 \right), \quad (12)$$

the chordal distance u is given by

$$u \equiv \frac{(z-w)^2}{2 z_0 w_0}, \quad (z-w)^2 = \delta_{CD} (z-w)_C (z-w)_D. \quad (13)$$

The function $G(u)$ above denotes the massless scalar propagator in AdS,

$$G(u) = \frac{\Gamma\left[\frac{d+1}{2}\right]}{d (4\pi)^{\frac{d+1}{2}}} (2u^{-1})^d {}_2F_1\left[d, \frac{d+1}{2}; d+1; -2u^{-1}\right]. \quad (14)$$

The function $H(u)$ sitting next to the other tensor structure is expressed as [30, 10]

$$H(u) = -\frac{1}{d-1} [2(1+u)^2 G(u) + 2(d-2) p'(u)], \quad (15)$$

$$p'(u) = \frac{-2}{d(d-1)} \frac{\Gamma\left[\frac{d+1}{2}\right]}{(4\pi)^{\frac{d+1}{2}}} (2u^{-1})^{d-1} {}_2F_1\left[d-1, \frac{d+1}{2}; d+1; -2u^{-1}\right]. \quad (16)$$

There exists an expression for the massless scalar propagator $G(u)$ in the split and embedding formalism that is suitable for Mellin-space computations [3, 4],

$$G(u) = \int \frac{dc}{2\pi i} f_0(c) \int_{\partial\text{AdS}} dQ \int d^2 s_c e^{2s Q \cdot X + 2\bar{s} Q \cdot Y},$$

$$d^2 s_c \equiv \frac{ds}{s} \frac{d\bar{s}}{\bar{s}} s^{h+c} \bar{s}^{h-c}, \quad f_0(c) \equiv \frac{1}{2\pi^2 h (h^2 - c^2)} \frac{1}{\Gamma[c] \Gamma[-c]}, \quad (17)$$

where we have introduced

$$h \equiv \frac{d}{2}. \quad (18)$$

It would be natural to use a formula of this type for the graviton bulk-to-bulk propagator in order to compute the Witten diagram of present interest. A candidate for this propagator has been proposed in [31]:

$$G_{A'B'}^{AB}(X; Y) = \int \frac{dc}{2\pi i} f_0(c) [(h+1)^2 - c^2] \int_{\partial\text{AdS}} dQ \int \frac{ds}{s} s^{h+c} (D_{h+c}^{MNAB} e^{2s Q \cdot X}) \times$$

$$\times \mathcal{E}_{MNPQ} \int \frac{d\bar{s}}{\bar{s}} \bar{s}^{h-c} (D_{h-c}^{PQ A'B'} e^{2\bar{s} Q \cdot Y}), \quad (19)$$

where

$$D_{\Delta}^{M_1 M_2 A_1 A_2} \equiv \eta^{M_1 A_1} \eta^{M_2 A_2} + \frac{1}{\Delta} (\eta^{M_1 A_1} P^{A_2} \partial_{M_2} + (1 \leftrightarrow 2)) + \frac{P^{A_1} P^{A_2}}{\Delta(\Delta+1)} \partial_{M_1} \partial_{M_2} \quad (20)$$

and

$$\mathcal{E}_{MNPQ} \equiv \frac{1}{2} (\eta_{MP} \eta_{NQ} + \eta_{MQ} \eta_{NP}) - \frac{1}{d} \eta_{MN} \eta_{PQ}, \quad (21)$$

with the metric components η_{MN} being the ones of a $d+2$ -dimensional Minkowski space, as defined in equation (7). This cannot be the right expression though. For instance, albeit

it correctly reproduces the poles (and their residues) entering the known expression for the Mellin transform of the Witten diagram between four minimally-coupled scalars in AdS_5 , it fails to yield the finite part accurately, namely the final piece $-\frac{5}{4}(3s_{12} - 22)$ in equation (44) below. The whole point is that (19) is traceless while the graviton propagator is not, actually.

An alternative embedding-formalism and split-formalism expression for the graviton propagator appears in [32]. Unfortunately, it turns out that this proposal is riddled with its own shortcomings and does not reproduce (44) either. A search for the correct formula is still on [33] but has not proved fruitful so far.

In view of those unexpected hindrances, any Mellin-transform of a Witten diagram involving the exchange of a graviton in the bulk of AdS_{d+1} will have to make direct use of the position-space formula (11). This introduces some factor of $X \cdot Y$, $P_i \cdot X$ or $P_j \cdot Y$. Transforming those stray X 's and Y 's into an expression appropriate to a Mellin-space formulation requires some care, much more than if an out-of-the-box split formula for the graviton propagator were available. Still, it is an entirely straightforward task and once rules for doing so have been written, they can be recycled for the computation of any Witten diagram, which is thereby reduced to a simple and automated process. In particular, it is possible to combine our result with previous evaluations of other Witten diagrams. That would thus provide the missing ingredient to wrap up the supergravity computation of the planar four-point Green's function for scalars of arbitrary scaling dimension.

Also of much interest, it should be possible to compute the four-point function of the $\mathcal{N} = 4$ sYM stress-tensor by that means [34]. The four-point Green's function of the stress-tensor is available for any two-dimensional CFT [35]; it is universal and depends only on the central charge c . Results have also been obtained fairly recently for three-dimensional CFT's with a supergravity dual [36] using new recursion relations for AdS/CFT correlators [37], which are reminiscent of those of Risager for perturbative field theories [38]. Clearly, an explicit $\text{AdS}_5/\text{CFT}_4$ calculation of this quantity is much sought-after⁴. Once again, the only valid reasons for not going ahead with such a computation mostly have to do with a bias against its presumed bulkiness and the expectation that a more concise and neater formula should be obtainable with even less effort if one were able to find a split expression for the bulk-to-bulk graviton propagator.

In the present work, such considerations of elegance have been temporarily swept aside. We are of the opinion that our universal formula for the 4-scalar graviton-exchange diagram is simple enough with regard to its universality, even though one might object that a snappier expression should be at hand using a, so far elusive, split formula for the graviton propagator similar to (17) for the scalar propagator. If anything, it illustrates how powerful a Mellin-space formulation of AdS/CFT correlation functions really is, subsuming and generalizing extensively more than a decade-worth of efforts where 4-point functions of scalar operators have been computed only for quite restricted sets of scaling dimensions.

Even though we do not provide in this note all the details of the method that leads to our universal formula⁵, it is worth emphasizing that we are making use of the split formalism to the best of what is currently achievable. First of all, we are inserting into (11) the split expression for the scalar propagator, namely equation (17). Furthermore, $H(u)$ itself can be

⁴See [39, 40, 41] for progress in this direction.

⁵A Mathematica notebook is available from the author.

cast into a much more convenient form. To do so, first of all, rewrite $H(u)$ by applying the second of Euler's hypergeometric transformations, i.e.

$${}_2F_1[a, b; c; z] = \frac{{}_2F_1\left[c - a, b; c; \frac{z}{z-1}\right]}{(1-z)^b}, \quad (22)$$

in order to express $p(u)'$ (16) in terms of $G(u)$ and its first-order derivative. We can then trade ${}_2F_1\left[2, \frac{d+1}{2}, d+1, \frac{\frac{-2}{u}}{\frac{-2}{u}-1}\right]$, which is proportional to $p(u)'$, for

$$\frac{\frac{\frac{-2}{u}}{\frac{-2}{u}-1}}{\left(\frac{\frac{-2}{u}}{\frac{-2}{u}-1}\right)'} \left(\frac{1}{\left(\frac{2}{u}\right)^d \left(1 + \frac{2}{u}\right)^{-\frac{d+1}{2}}} G(u) \right)' + \frac{1}{\left(\frac{2}{u}\right)^d \left(1 + \frac{2}{u}\right)^{-\frac{d+1}{2}}} G(u). \quad (23)$$

This results in

$$H(u) = -2u^2 G(u) - \frac{2(2-d)u(1-u^2)G'(u)}{(1-d)^2}. \quad (24)$$

On the other hand⁶,

$$G(u) = \int \frac{dc}{2\pi i} f_0(c) D_c(u), \quad D_c(u) \equiv \int_{\partial \text{AdS}} dQ \frac{|X|^{h+c}}{(-2X \cdot Q)^{h+c}} \frac{|Y|^{h-c}}{(-2Y \cdot Q)^{h-c}}. \quad (25)$$

For any function of the chordal distance u ⁷, one can verify that

$$\frac{\partial}{\partial u} f(u) = \frac{1}{1-u^2} \frac{|X|}{|Y|} Y^M \frac{\partial}{\partial X^M} f, \quad (26)$$

so that in particular

$$D_c(u)' = \frac{h^2 - c^2}{2h(-1+u^2)} \left[D_{c+1}(u) + D_{c-1}(u) - 2u D_c(u) \right], \quad (27)$$

using the fact that $D_c(u) = D_{-c}(u)$. All in all, combining (24), (25) and (27) brings in an expression for $H(u)$ similar to the one for the scalar propagator $G(u)$ in the split and embedding formalism, equation (17).

⁶The factors of $|X \cdot X|^{\frac{h+c}{2}}$ and $|Y \cdot Y|^{\frac{h-c}{2}}$ are there to enforce the conditions $X^2 = -R^2$ and $Y^2 = -R^2$ characterizing the embedding of an AdS_{d+1} space of radius R into a $d+2$ -dimensional Minkowski space. As explained in Section 2, this condition is central to the Mellin-space formulation of AdS/CFT correlation functions.

⁷From now on, we are going to define the chordal distance as $u = -X \cdot Y$. We have taken this into account when massaging the bulk-to-bulk propagator (11) where u is actually $u_D = -1 + u$ with our new convention.

4 Evaluating the 4-scalar graviton-exchange Witten diagram

The Mellin amplitude for the 4-scalar graviton-exchange Witten diagram is quite unwieldy when the masses of the four external scalars are left unspecified. Henceforth, to keep this note relatively short, in this section we refrain from writing down the generic expression. The reader is referred to the Mathematica notebook accompanying this paper for the details of the most general formula. Another Mathematica notebook is available upon request, where this end-product is derived step by step in a way that should also prove useful for computing other contributions to correlations functions at strong coupling.

For illustrative purposes, however, here we provide the Mellin amplitude for the Witten diagram corresponding to the exchange of a graviton in AdS_{d+1} between scalars of masses $m_1^2 = d_1(d_1 - d)$ and $m_2^2 = d_2(d_2 - d)$, where the space-time dimension $d \equiv 2h$ of the boundary is further set to $d = 4$. According to the AdS/CFT dictionary, the d_i 's correspond to the conformal weights of the dual operators.

Quite generally, our universal formula is expressed in terms of the following quantities, which are proportional to the Mellin-space formulation of scalar-exchange Witten diagrams:

$$\begin{aligned} \Upsilon[\Delta_1, \Delta_2; \delta; z] &\equiv \frac{\Gamma[\Delta_1 + \frac{1}{2}(\delta - 4)] \Gamma[\Delta_2 + \frac{1}{2}(\delta - 4)]}{\Gamma[\delta - 1](z - \delta)} \times \\ &\times {}_3F_2\left[1 - \Delta_1 + \frac{\delta}{2}, 1 - \Delta_2 + \frac{\delta}{2}, \frac{\delta - z}{2}; \delta - 1, 1 + \frac{\delta - z}{2}; 1\right]. \end{aligned} \quad (28)$$

Indeed,

$$\begin{aligned} \Upsilon[\Delta_1, \Delta_2; \delta; z] &= \int \frac{dc}{2\pi i} \frac{1}{\Gamma[\Delta_1 - \frac{z}{2}] \Gamma[\Delta_2 - \frac{z}{2}]} \Gamma\left[1 + \frac{1}{2}(c - z)\right] \Gamma\left[1 - \frac{1}{2}(c + z)\right] \times \\ &\times \frac{\Gamma[\Delta_1 - 1 + \frac{c}{2}] \Gamma[\Delta_1 - 1 - \frac{c}{2}] \Gamma[\Delta_2 - 1 + \frac{c}{2}] \Gamma[\Delta_2 - 1 - \frac{c}{2}]}{4(c^2 - (\delta - 2)^2) \Gamma[c] \Gamma[-c]}. \end{aligned} \quad (29)$$

Starting with our universal formula for the Mellin amplitude $\mathcal{M}(d_1, d_2, d_3, d_4; \gamma_{12}, s_{12}; h)$ of the Witten diagram corresponding to the exchange of a graviton in AdS_{2h+1} between four bulk scalars of respective mass-squared $m_i^2 = d_i(d_i - 2h)$, we can particularize it to $h = 2$, $d_3 = d_1$ and $d_4 = d_2$. This leads to the following expression, spelt out in terms of the “Mandelstam invariant” s_{12} (cf. (6)) and the variable

$$\gamma_{12} \equiv \frac{1}{2}(s_{13} - s_{23}). \quad (30)$$

This particular case of our general expression for the Mellin amplitude of a 4-scalar graviton-

exchange Witten diagram takes on the following form:

$$\begin{aligned}
\mathcal{M}(d_1, d_2; \gamma_{12}, s_{12}) = & \mathcal{P}_{(2,1)} \Upsilon\left[\frac{1}{2}(d_1 + d_2) + 2, \frac{1}{2}(d_1 + d_2) + 1; 4; s_{12} + 2\right] \\
& + \mathcal{P}_{(1,1)} \Upsilon\left[\frac{1}{2}(d_1 + d_2) + 1, \frac{1}{2}(d_1 + d_2) + 1; 4; s_{12} + 2\right] \\
& + \mathcal{P}_{(1,0)} \Upsilon\left[\frac{1}{2}(d_1 + d_2) + 1, \frac{1}{2}(d_1 + d_2); 4; s_{12} + 2\right] \\
& + \mathcal{P}_{(0,0)} \Upsilon\left[\frac{1}{2}(d_1 + d_2), \frac{1}{2}(d_1 + d_2); 4; s_{12} + 2\right] \\
& + \mathcal{P}_{(0,-1)} \Upsilon\left[\frac{1}{2}(d_1 + d_2), \frac{1}{2}(d_1 + d_2) - 1; 4; s_{12} + 2\right] \\
& + \mathcal{P}_{(-1,-1)} \Upsilon\left[\frac{1}{2}(d_1 + d_2) - 1, \frac{1}{2}(d_1 + d_2) - 1; 4; s_{12} + 2\right] \\
& + \mathcal{P}_{\frac{3}{2}, \frac{3}{2}} \Upsilon\left[\frac{1}{2}(3 + d_1 + d_2), \frac{1}{2}(3 + d_1 + d_2); 3; s_{12} + 1\right] \\
& + \mathcal{P}_{\frac{3}{2}, \frac{1}{2}} \Upsilon\left[\frac{1}{2}(3 + d_1 + d_2), \frac{1}{2}(1 + d_1 + d_2); 3; s_{12} + 1\right] \\
& + \mathcal{P}_{\frac{1}{2}, \frac{1}{2}} \Upsilon\left[\frac{1}{2}(1 + d_1 + d_2), \frac{1}{2}(1 + d_1 + d_2); 3; s_{12} + 1\right] \\
& + \mathcal{P}_{\frac{1}{2}, -\frac{1}{2}} \Upsilon\left[\frac{1}{2}(1 + d_1 + d_2), \frac{1}{2}(-1 + d_1 + d_2); 3; s_{12} + 1\right] \\
& + \mathcal{P}_{-\frac{1}{2}, -\frac{1}{2}} \Upsilon\left[\frac{1}{2}(-1 + d_1 + d_2), \frac{1}{2}(-1 + d_1 + d_2); 3; s_{12} + 1\right], \quad (31)
\end{aligned}$$

where the

$$\mathcal{P}_{(i,j)} = \mathcal{P}_{(i,j)}[d_1, d_2; \gamma_{12}, s_{12}] \quad (32)$$

are given by

$$\begin{aligned}
\mathcal{P}_{(2,1)} = & -\frac{\pi^2}{16} (d_1 + d_2 - s_{12}) \left[8 + 4d_1^2 - 10d_2 + 2d_1(-5 + 4d_2 - 2s_{12}) \right. \\
& \left. + 5s_{12} + (-2d_2 + s_{12})^2 \right], \quad (33)
\end{aligned}$$

$$\begin{aligned}
\mathcal{P}_{(1,1)} = & \frac{\pi^2}{96} \left\{ 7120 - 6192d_2 + 4(9d_1^4 + d_1^3[-45 + 28d_2] \right. \\
& + d_2^2(422 + 9[-5 + d_2]d_2) + d_1^2(422 + d_2[-99 + 38d_2]) \\
& + d_1[-1548 + d_2(798 + d_2[-99 + 28d_2])]) \\
& + 3084s_{12} - s_{12}[48d_1^3 + d_1^2[-195 + 112d_2] \\
& + d_2(1616 + 3d_2[-65 + 16d_2]) + 2d_1(808 + d_2[-159 + 56d_2])] \\
& \left. + (359 + 15d_1^2 + 3d_2[-14 + 5d_2] + d_1[-42 + 22d_2])s_{12}^2 - 12\gamma_{12}^2 \right\}, \quad (34)
\end{aligned}$$

$$\begin{aligned}
\mathcal{P}_{(1,0)} = & -\frac{\pi^2}{288 (-2 + d_1 + d_2 - s_{12})} \left\{ 54 d_1^6 + 54 d_2^6 + 9 d_1^5 (-61 + 20 d_2 - 10 s_{12}) \right. \\
& - 9 d_2^5 (61 + 10 s_{12}) + 9 d_2^4 (907 + 90 s_{12} + 4 s_{12}^2) + d_1^3 (-61404 + 2 d_2 (12406 + d_2 [-761 + 140 d_2]) \\
& - 4 (2694 + d_2 [-447 + 89 d_2]) s_{12} + 3 [-93 + 28 d_2] s_{12}^2) - 3 d_2^3 (20468 + s_{12} [3592 + 93 s_{12}]) \\
& - 8 d_2 (88220 + s_{12} [26578 + 1713 s_{12}]) + 4 (199224 + s_{12} [77546 + 7577 s_{12}]) \\
& + d_1^4 (266 d_2^2 - 3 d_2 [451 + 86 s_{12}] + 9 [907 + 90 s_{12} + 4 s_{12}^2]) + 2 d_2^2 (2 (67715 + 3 s_{12} [5353 + 272 s_{12}]) \\
& - 9 [-2 + s_{12}] \gamma_{12}) + d_1^2 (266 d_2^4 - 2 d_2^3 [761 + 178 s_{12}] + 2 d_2^2 (16817 + 2 s_{12} [515 + 26 s_{12}]) \\
& - 5 d_2 (31700 + s_{12} [5176 + 105 s_{12}]) + 6 s_{12} (10706 + 544 s_{12} - 3 \gamma_{12}) + 4 [67715 + 9 \gamma_{12}]) \\
& + d_1 [180 d_2^5 - 3 d_2^4 (451 + 86 s_{12}) + 4 d_2^3 (6203 + 3 s_{12} [149 + 7 s_{12}]) \\
& - 5 d_2^2 (31700 + s_{12} [5176 + 105 s_{12}]) - 8 (88220 + s_{12} [26578 + 1713 s_{12}]) + 4 d_2 (9 [-2 + s_{12}] \gamma_{12} \\
& \left. + 2 (63515 + s_{12} [14569 + 685 s_{12}])) \right\}, \tag{35}
\end{aligned}$$

$$\begin{aligned}
\mathcal{P}_{(0,0)} = & \frac{\pi^2}{288 (-2 + d_1 + d_2 - s_{12})^2} \left\{ 9 d_1^8 + 9 d_2^8 - 18 d_1^7 (7 + s_{12}) - 18 d_2^7 (7 + s_{12}) \right. \\
& + 9 d_2^6 (407 + s_{12} [26 + s_{12}]) + 96 (117888 + s_{12} [56986 + 5901 s_{12}]) \\
& - 8 d_2 (1822776 + s_{12} [683202 + 55205 s_{12}]) + 3 d_1^6 [-44 d_2^2 - 2 d_2 (-47 + s_{12}) + 3 (407 + s_{12} [26 + s_{12}])] \\
& - 3 d_2^5 (16468 + s_{12} (2053 + 36 s_{12}) + 6 \gamma_{12}) - d_1^5 (49404 + 6 d_2 (-1659 + d_2 [-425 + 64 d_2]) + 6159 s_{12} \\
& + 2 (138 - 79 d_2) d_2 s_{12} - 6 (-18 + d_2) s_{12}^2 + 18 \gamma_{12}) + 4 d_2^2 (1947468 + 561902 s_{12} + 34925 s_{12}^2 \\
& + 36 (2 + s_{12}) \gamma_{12}) + 6 d_2^4 (68524 + 11971 s_{12} + 416 s_{12}^2 + 3 (8 + s_{12}) \gamma_{12}) \\
& - 4 d_2^3 (568704 + 90 \gamma_{12} + s_{12} [127018 + 6081 s_{12} + 27 \gamma_{12}]) \\
& + d_1^2 (-132 d_2^6 + 2 d_2^5 (1275 + 79 s_{12}) + d_2^4 (9969 - s_{12} [2566 + 37 s_{12}]) + 2 d_2^3 (s_{12} (-9411 + 238 s_{12}) \\
& + 18 [-7130 + \gamma_{12}]) - 4 d_2 (1545696 + s_{12} (324414 + 14107 s_{12} - 27 \gamma_{12}) - 90 \gamma_{12}) \\
& + 4 (1947468 + 561902 s_{12} + 34925 s_{12}^2 + 36 (2 + s_{12}) \gamma_{12}) + 4 d_2^2 [459036 + 67615 s_{12} + 1690 s_{12}^2 \\
& - 9 (8 + s_{12}) \gamma_{12}]) + d_1^4 [-522 d_2^4 + 6 d_2^3 (893 + 63 s_{12}) + d_2^2 (9969 - s_{12} [2566 + 37 s_{12}]) \\
& + 6 (68524 + 11971 s_{12} + 416 s_{12}^2 + 3 (8 + s_{12}) \gamma_{12}) + d_2 (s_{12} (-15787 + 24 s_{12}) - 18 [9198 + \gamma_{12}])] \\
& + d_1 [-6 d_2^6 (-47 + s_{12}) + 6 d_2^5 (1659 + (-46 + s_{12}) s_{12}) + 8 d_2^3 (165792 + s_{12} [25705 + 721 s_{12}]) \\
& - 8 (1822776 + s_{12} [683202 + 55205 s_{12}]) - 4 d_2^2 (1545696 + s_{12} (324414 + 14107 s_{12} - 27 \gamma_{12}) - 90 \gamma_{12}) \\
& + 8 d_2 (1882956 + 531278 s_{12} + 31901 s_{12}^2 - 36 (2 + s_{12}) \gamma_{12}) \\
& + d_2^4 (s_{12} (-15787 + 24 s_{12}) - 18 [9198 + \gamma_{12}])] + 2 d_1^3 [-192 d_2^5 + 3 d_2^4 (893 + 63 s_{12}) \\
& + d_2^3 (3678 - 2 s_{12} [1028 + 17 s_{12}]) + 4 d_2 (165792 + s_{12} [25705 + 721 s_{12}]) + d_2^2 (s_{12} (-9411 + 238 s_{12}) \\
& \left. + 18 [-7130 + \gamma_{12}]) - 2 (568704 + 90 \gamma_{12} + s_{12} [127018 + 6081 s_{12} + 27 \gamma_{12}]) \right\}, \tag{36}
\end{aligned}$$

$$\begin{aligned}
\mathcal{P}_{(0,-1)} = & \frac{\pi^2}{72} \frac{-3 + d_1 + d_2}{(-4 + d_1 + d_2 - s_{12}) (-2 + d_1 + d_2 - s_{12})^2} \left\{ 3 d_1^8 d_2 \right. \\
& + 3 d_1^7 (-40 + d_2 [-25 + 5 d_2 - 2 s_{12}]) + d_1^5 (-41952 + 3892 d_2 + 3068 d_2^2 - 665 d_2^3 \\
& + 45 d_2^4 - 2 (2640 + d_2 [-43 + 5 d_2] [-7 + 5 d_2]) s_{12} + (-120 + d_2 [-57 + 11 d_2]) s_{12}^2 \\
& + d_1^4 (d_2 (-121312 + d_2 [-7988 + 9 d_2 (676 + 5 (-18 + d_2) d_2)]) \\
& - 2 d_2 (3698 + d_2 [1783 + 30 (-14 + d_2) d_2]) s_{12} + [2280 + d_2 (170 + 3 d_2 [-59 + 6 d_2])]) s_{12}^2 \\
& + 208 [1816 + 313 s_{12}]) + d_1^2 (d_2 (-5773568 + d_2 [1648256 + d_2 (-160688 \\
& + d_2 [-7988 + d_2 (3068 + 5 d_2 [-67 + 3 d_2])])]) + 2 d_2 (-630320 + d_2 [101316 + d_2 (694 \\
& + d_2 [-1783 + (250 - 13 d_2) d_2])]) s_{12} + [150816 + d_2 (-50992 + d_2 [2428 + d_2 (866 \\
& + d_2 [-177 + 11 d_2])])]) s_{12}^2 + 960 [7302 + 2347 s_{12}]) + 8 (d_2 [-1563136 + d_2 (876240 \\
& + d_2 [-264416 + d_2 (47216 - 3 d_2 [1748 + 5 (-25 + d_2) d_2])])]) \\
& + 2 d_2 (-335896 + d_2 [140820 + d_2 (-31294 + d_2 [4069 + 15 (-22 + d_2) d_2])]) s_{12} \\
& + (81536 + d_2 [-61736 + d_2 (18852 + d_2 [-3014 - 15 (-19 + d_2) d_2])]) s_{12}^2 + 256 [4483 + 2556 s_{12}]) \\
& + d_1^3 (d_2 [1199488 + d_2 (-160688 + d_2 [-17760 + d_2 (6084 + d_2 [-665 + 33 d_2])])]) \\
& + 2 d_2 (82472 + d_2 [694 + d_2 (-2724 + 5 (84 - 5 d_2) d_2)]) s_{12} + 2 (-12056 + d_2 [1644 \\
& + d_2 (433 + 3 d_2 [-40 + 3 d_2])]) s_{12}^2 - 32 [66104 + 15647 s_{12}]) + d_1 (d_2 (13594880 \\
& + d_2 [-5773568 + d_2 (1199488 + d_2 [-121312 + d_2 (3892 + 3 d_2 [152 + (-25 + d_2) d_2])])]) \\
& - 2 d_2 [-2132160 + d_2 (630320 + d_2 [-82472 + d_2 (3698 + d_2 [301 + 3 (-22 + d_2) d_2])])]) s_{12} \\
& + [-493888 + d_2 (271552 + d_2 [-50992 + d_2 (3288 + d_2 [170 + 3 (-19 + d_2) d_2])])]) s_{12}^2 \\
& - 128 [97696 + 41987 s_{12}]) + d_1^6 [33 d_2^3 + 120 (25 + 2 s_{12}) - d_2^2 (335 + 26 s_{12}) \\
& + 3 d_2 (152 + s_{12} [44 + s_{12}])]) \left. \right\}, \tag{37}
\end{aligned}$$

$$\begin{aligned}
\mathcal{P}_{(-1,-1)} = & \frac{\pi^2}{72} \frac{(-5 + d_1 + d_2) (-4 + d_1 + d_2)^2 (-3 + d_1 + d_2)}{(-4 + d_1 + d_2 - s_{12}) (-2 + d_1 + d_2 - s_{12})} \times \\
& \times \left\{ d_1^4 d_2^2 + 64 (516 + 25 [-8 + d_2] d_2) + 2 d_1^3 d_2 (-40 + [-4 + d_2] d_2) \right. \\
& - 16 d_1 [800 + d_2 (-104 + 5 [-8 + d_2] d_2)] \\
& \left. + d_1^2 (1600 + d_2 [640 + d_2 (-148 + [-8 + d_2] d_2)]) \right\}, \tag{38}
\end{aligned}$$

$$\mathcal{P}_{(\frac{3}{2}, \frac{3}{2})} = -\frac{\pi^2}{16} (d_1 + d_2 - s_{12})^2 (2 [1 - d_1 - d_2] + s_{12}), \tag{39}$$

$$\begin{aligned}
\mathcal{P}_{\frac{3}{2}, \frac{1}{2}} = & -\frac{\pi^2}{96} (d_1 + d_2 - s_{12})^2 \left\{ 4 (-329 + 3 d_1^3 + d_1^2 (-9 + 5 d_2) \right. \\
& + d_2 (175 + 3 (-3 + d_2) d_2) + 5 d_1 [35 + (-2 + d_2) d_2]) \\
& \left. - (350 + 9 d_1^2 + 3 d_2 (-8 + 3 d_2) + 2 d_1 (-12 + 5 d_2)) s_{12} \right\}, \tag{40}
\end{aligned}$$

$$\begin{aligned} \mathcal{P}_{\frac{1}{2}, \frac{1}{2}} = & \frac{\pi^2}{288} \left\{ 9 d_1^5 + d_1^3 (2046 + 2 (54 - 19 d_2) d_2 + 45 s_{12}) - 4 (25920 + 5867 s_{12}) \right. \\ & - 3 d_1^4 (d_2 + 3 [5 + s_{12}]) + d_1^2 (-54 (217 + 29 s_{12}) + d_2 [4138 - 33 s_{12} + 2 d_2 (129 - 19 d_2 + 5 s_{12})]) \\ & + d_2 (54508 + 6918 s_{12} + 3 d_2 [d_2 (682 + 3 d_2 (-5 + d_2 - s_{12}) + 15 s_{12}) - 18 (217 + 29 s_{12})]) \\ & \left. + d_1 (54508 + 6918 s_{12} - d_2 [d_2 (-4138 + 3 (-36 + d_2) d_2 + 33 s_{12}) + 4 (4731 + 557 s_{12})]) \right\}, \end{aligned} \quad (41)$$

$$\begin{aligned} \mathcal{P}_{\frac{1}{2}, -\frac{1}{2}} = & \frac{\pi^2}{72} \frac{(40 - d_1 d_2)}{(2 - d_1 - d_2 + s_{12})} \left\{ 3 d_1^5 - 64 (93 + 29 s_{12}) + d_1^3 (406 + 3 d_2 (-44 + 6 d_2 - 3 s_{12}) + 36 s_{12}) \right. \\ & + d_1^4 (11 d_2 - 3 [14 + s_{12}]) + 2 d_1^2 (-21 (52 + 7 s_{12}) + d_2 [551 + 9 d_2^2 + 44 s_{12} - 6 d_2 (15 + s_{12})]) \\ & + d_2 (5768 + 1196 s_{12} + d_2 (-42 (52 + 7 s_{12}) + d_2 (406 + 3 d_2 (-14 + d_2 - s_{12}) + 36 s_{12}))) \\ & \left. + d_1 (5768 + 1196 s_{12} + d_2 (-4236 - 550 s_{12} + d_2 (1102 + d_2 (-132 + 11 d_2 - 9 s_{12}) + 88 s_{12}))) \right\} \end{aligned} \quad (42)$$

and at long last

$$\begin{aligned} \mathcal{P}_{-\frac{1}{2}, -\frac{1}{2}} = & \frac{\pi^2}{72} (-4 + d_1 + d_2) (-3 + d_1 + d_2) (-40 + d_1 d_2)^2 \times \\ & \times \frac{(14 + d_1^2 + 2 d_1 [-3 + d_2] + [-6 + d_2] d_2)}{(-2 + d_1 + d_2 - s_{12})}. \end{aligned} \quad (43)$$

In particular, when all the d_i 's are set to $d_i = \Delta = 4$, we recover⁸ up to an overall normalization factor the known expression for the graviton-exchange between minimally-coupled scalars in AdS₅:

$$\mathcal{M}(\gamma_{12}, s_{12}) = \frac{6 \gamma_{12}^2 + 2}{s_{12} - 2} + \frac{\gamma_{12}^2 - 1}{s_{12} - 6} + \frac{8 \gamma_{12}^2}{s_{12} - 4} - \frac{5}{4} (3 s_{12} - 22). \quad (44)$$

This is in perfect agreement with equation (8) from reference [3] ! It is a non-trivial test on the validity of our universal formula. Indeed, in [3] the Mellin amplitude (44) is obtained by directly translating to Mellin-space a result first derived in position-space by entirely different means and expressed in terms of D-functions by D'Hoker and collaborators [10].

Our universal formula — to be found in the Mathematica notebook enclosed in the source for the arXiv version of this paper — can be used to evaluate in no time the Witten diagram describing the exchange of a graviton between scalars of arbitrary masses in AdS. From a dual perspective, this is the stress-tensor contribution to the four-point function of scalar operators at strong coupling, whatever their conformal weights and the dimension of the space in which they are defined. For instance, the graviton-exchange between minimally-coupled scalars, now in AdS₇, takes on the following form:

$$\frac{360 \pi (4 + 5 \gamma_{12}^2)}{s_{12} - 4} + \frac{810 \pi (3 + 5 \gamma_{12}^2)}{s_{12} - 6} + \frac{1620 \pi \gamma_{12}^2}{s_{12} - 8} - \frac{90 \pi (1 - \gamma_{12}^2)}{s_{12} - 10} + 1890 \pi (10 - s_{12}). \quad (45)$$

⁸See the affixed Mathematica file.

As another example among endless possible combinations, here is the AdS_5 graviton–exchange contribution to the CFT–dual Green’s function involving external scalar operators of respective dimensions $d_1 = 3$, $d_2 = 4$, $d_3 = 5$ and $d_4 = 7$:

$$\begin{aligned}
& \frac{2023 \sqrt{\pi} + 2250 \sqrt{\pi} \gamma_{12} + 6300 \sqrt{\pi} \gamma_{12}^2}{128 (s_{12} - 2)} + \frac{17251 \sqrt{\pi} + 1950 \sqrt{\pi} \gamma_{12} + 25200 \sqrt{\pi} \gamma_{12}^2}{256 (s_{12} - 4)} \\
& + \frac{456889 \sqrt{\pi} - 3825 \sqrt{\pi} \gamma_{12} + 18900 \sqrt{\pi} \gamma_{12}^2}{1024 (s_{12} - 6)} + \frac{-356779 \sqrt{\pi} + 2430 \sqrt{\pi} \gamma_{12} - 5040 \sqrt{\pi} \gamma_{12}^2}{4096 (s_{12} - 8)} \\
& + \frac{3 (59023 \sqrt{\pi} - 320 \sqrt{\pi} \gamma_{12} + 420 \sqrt{\pi} \gamma_{12}^2)}{16384 (s_{12} - 10)} + \frac{225225 \sqrt{\pi} (44 - 3 s_{12})}{16384}. \tag{46}
\end{aligned}$$

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